Introduction to Polarisation

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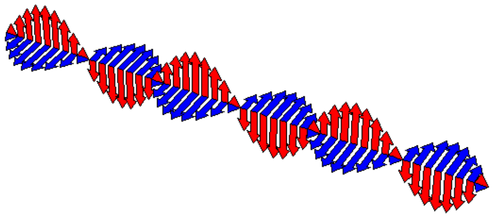
IPTA 2015 Student Workshop
Parkes Observatory, 22 July 2015

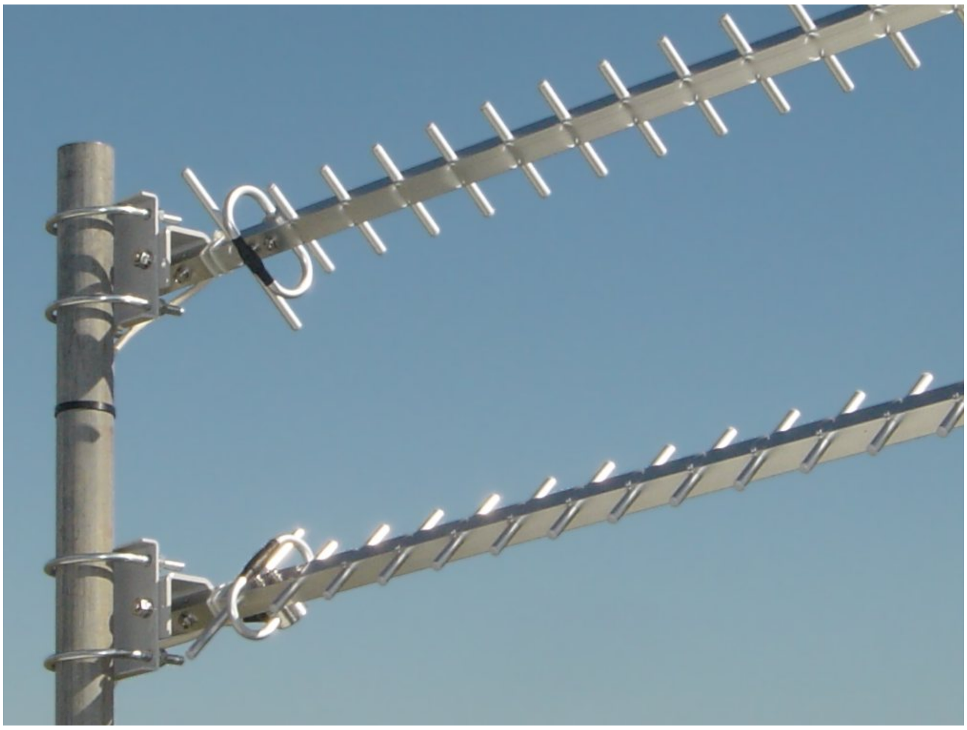
Outline

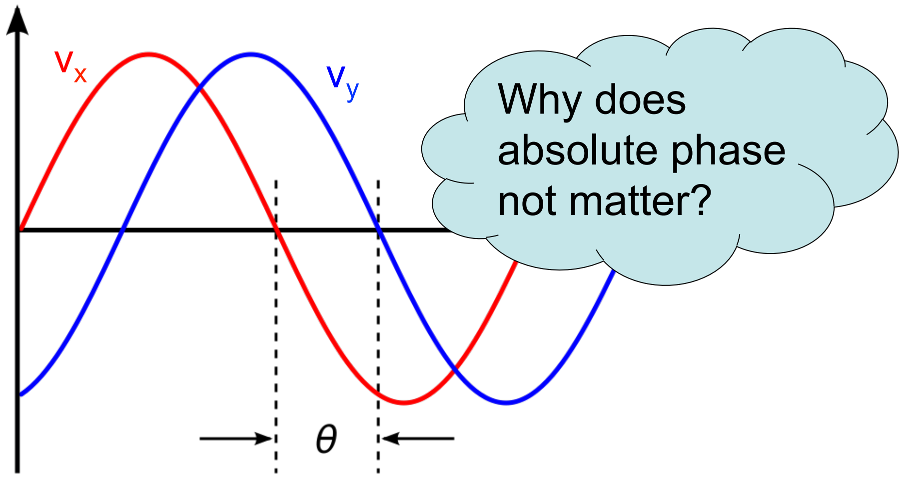
- Representations of polarisation
  - geometric and statistical
- Radio pulsar polarisation
  - rotating vector model
  - orthogonally polarized modes
  - high-precision timing
- Polarimetric calibration

Electromagnetic Plane Wave



Idealized: monochromatic



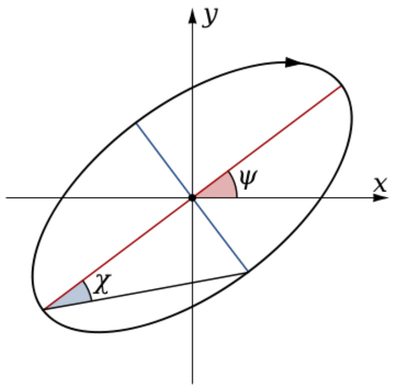


- 3 degrees of freedom
  - Amplitudes of v_x & v_y
  - Relative phase θ

Simple Demo

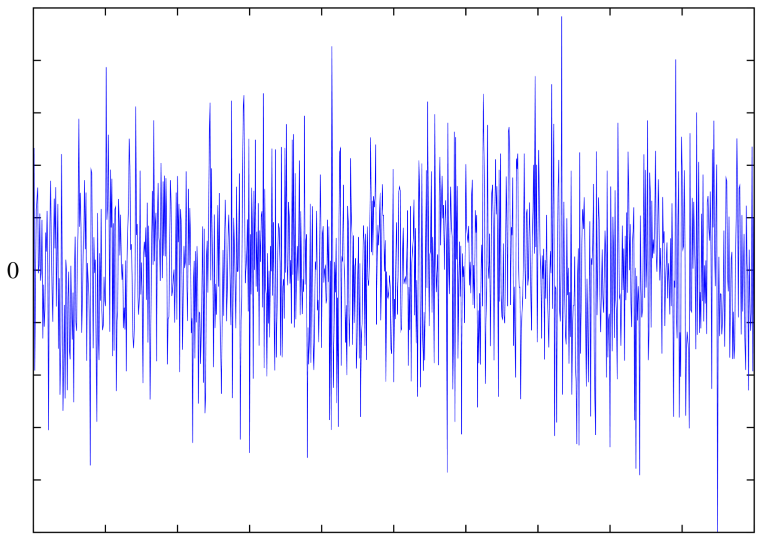
http://webphysics.davidson.edu/physlet_resources/dav_optics/examples/polarization.html

Polarisation Ellipse



- 3 degrees of freedom
  - size
  - orientation ψ,
  - axial ratio tanχ
    (direction = sign of χ)
- insensitive to absolute phase

Partially polarised light



Coherency Matrix

Variance of e_x

Covariance

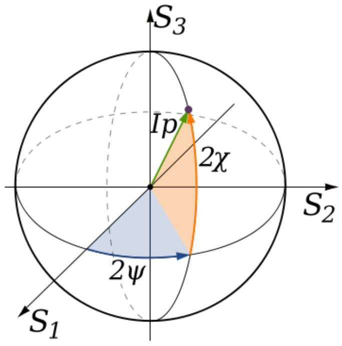
$$\boldsymbol{\rho} \equiv \langle \mathbf{e}\, \mathbf{e}^{\dagger} \rangle = \begin{pmatrix} \langle e_0 e_0^* \rangle & \langle e_0 e_1^* \rangle \\ \langle e_1 e_0^* \rangle & \langle e_1 e_1^* \rangle \end{pmatrix}$$

Variance of e_y

$$\boldsymbol{\rho} = \boldsymbol{\rho}^{\dagger}$$

- $<e_x\, e_x^*> + <e_y\, e_y^*> =$ **S_0** (total intensity)
- $<e_x\, e_x^*> - <e_y\, e_y^*> =$ **S_1** (can be non-zero)
- $<e_y\, e_x^*> = <a_y\, a_x \exp(j\varphi_y - j\varphi_x)>$
  $= <a_y\, a_x [\cos(\Delta\varphi) + j\sin(\Delta\varphi)]>$
- $<e_y\, e_x^*> =$ **S_2** $+ j$ **S_3** will be
  - zero when x and y are uncorrelated
  - purely real when $\Delta\varphi = 0$ or π
  - purely imaginary when $\Delta\varphi = \pi/2$ or $-\pi/2$

Stokes Parameters - 4 vector

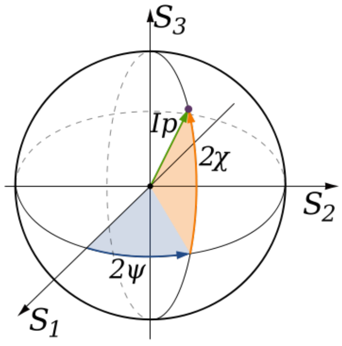


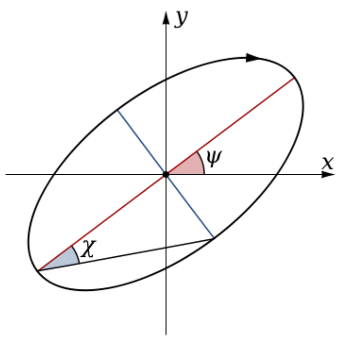
$S_0 = I$ total intensity
$\mathbf{S} = (S_1,S_2,S_3)$ polarisation vector

All have units of intensity

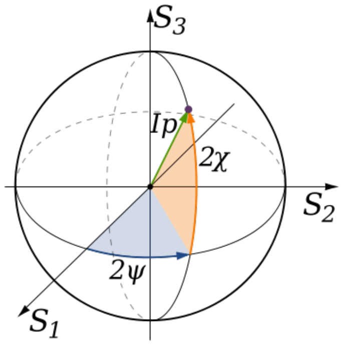
$p = |\mathbf{S}|/I$ degree of polarisation

Stokes Parameters - Ellipse





Stokes Parameters - Intensities



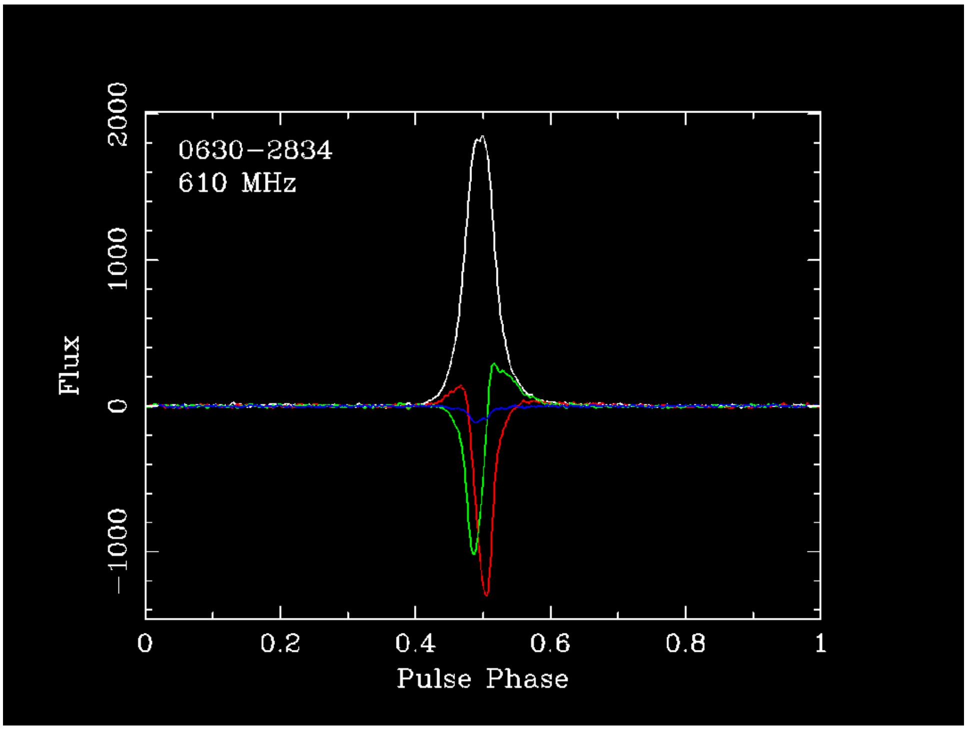
$$S_1 = \lambda_{\psi=0} - \lambda_{\psi=90}$$
$$S_2 = \lambda_{\psi=45} - \lambda_{\psi=135}$$
$$S_3 = \lambda_{\chi=45} - \lambda_{\chi=-45}$$

Stokes Parameters - Relation

$$\boldsymbol{\rho} = S_k\, \boldsymbol{\sigma}_k/2$$
$$S_k = \mathrm{Tr}(\boldsymbol{\sigma}_k \boldsymbol{\rho})$$

$$\boldsymbol{\sigma}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \boldsymbol{\sigma}_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$
$$\boldsymbol{\sigma}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \boldsymbol{\sigma}_3 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

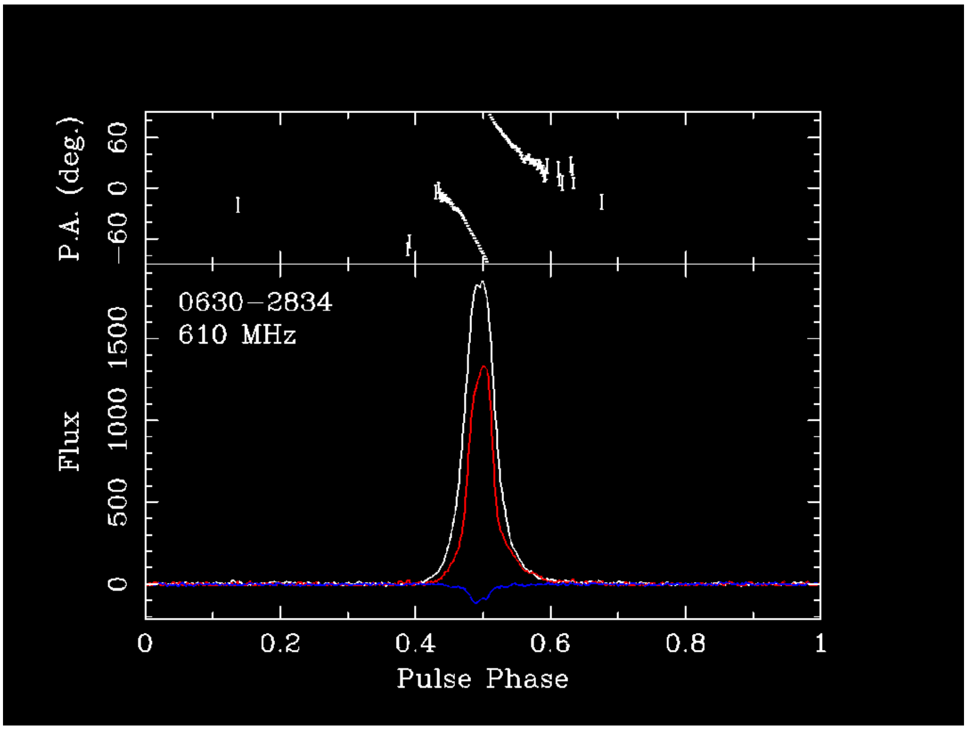
Rotating Vector Model

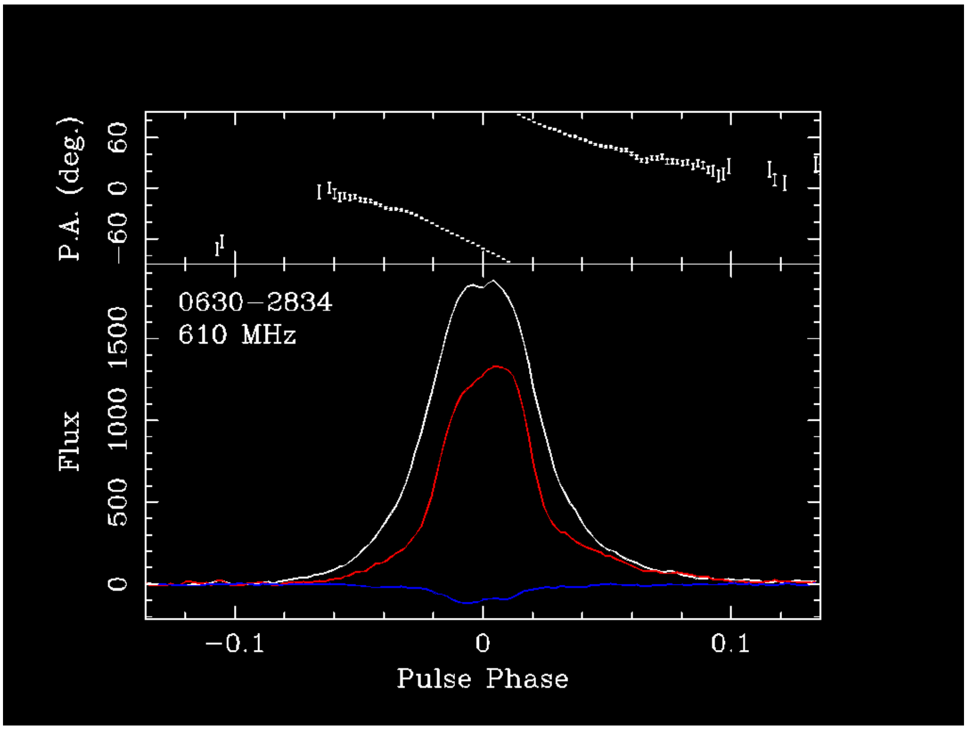


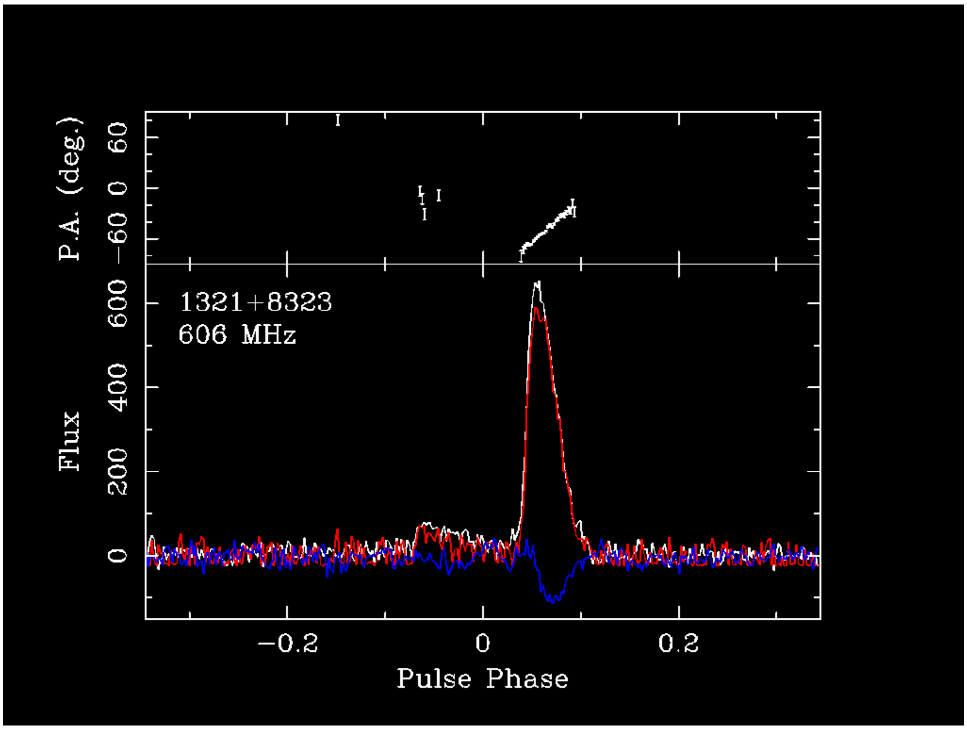
Stokes Parameters - Cylindrical

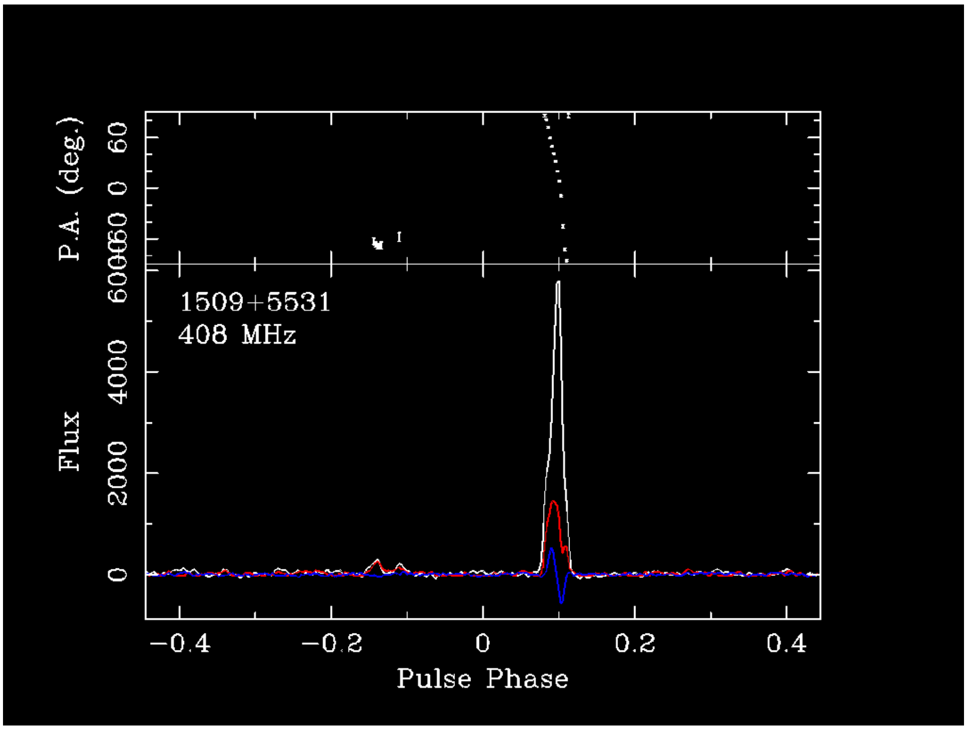
$$L = \sqrt{Q^2 + U^2}$$

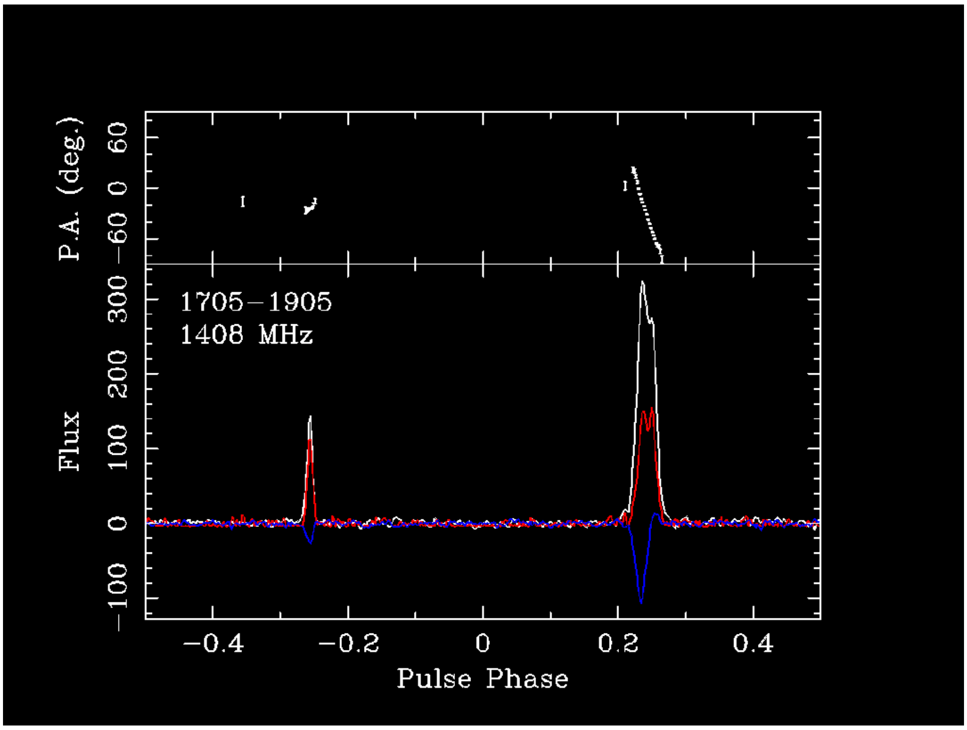
$$\psi = \frac{1}{2} \tan^{-1} \frac{U}{Q}$$




P.A. (deg.)
−60
0
60
Flux
0
500
1000
1500
0630−2834
610 MHz
−0.1
0
0.1
Pulse Phase

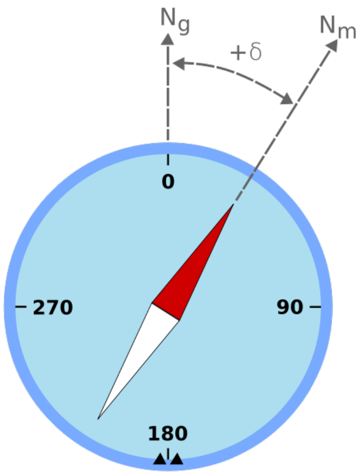

P.A. (deg.)
−60
0
60
Flux
0
200
400
600
1321+8323
606 MHz
−0.2
0
0.2
Pulse Phase

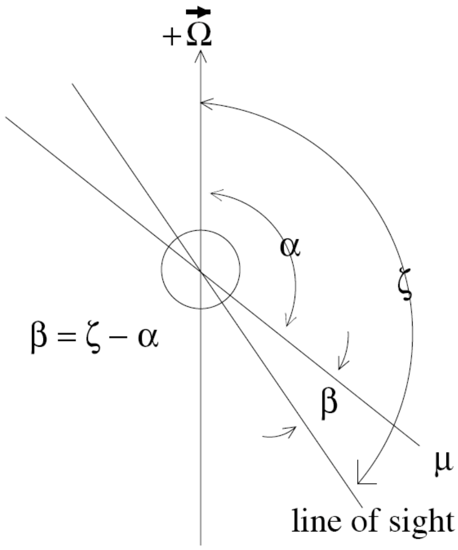

P.A. (deg.)
Flux
1509+5531
408 MHz
Pulse Phase


P.A. (deg.)
Flux
1705−1905
1408 MHz
Pulse Phase

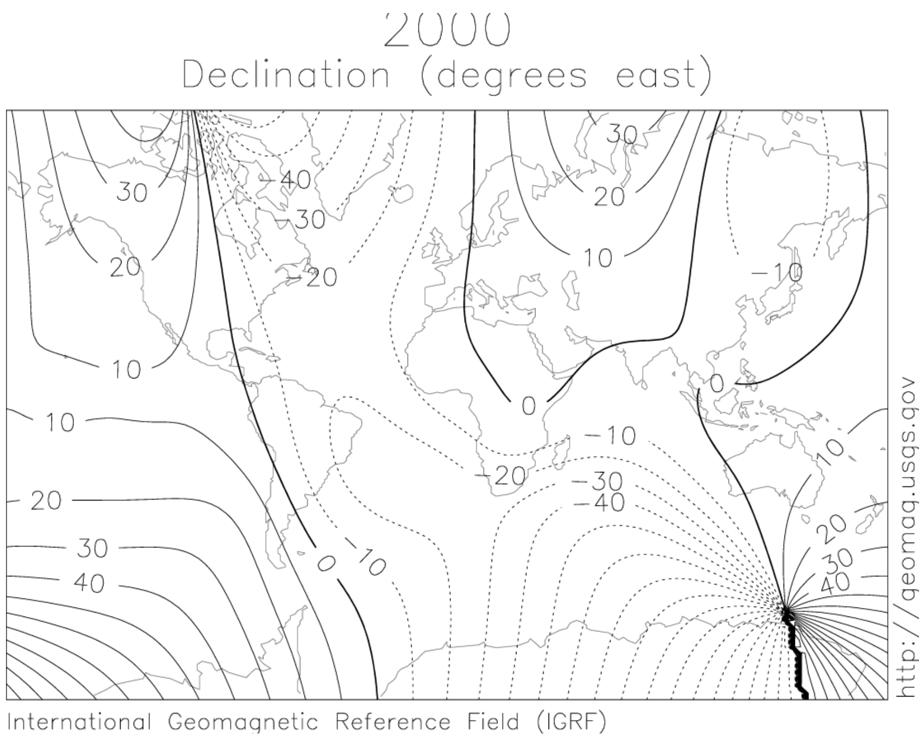
Rotating Vector Model

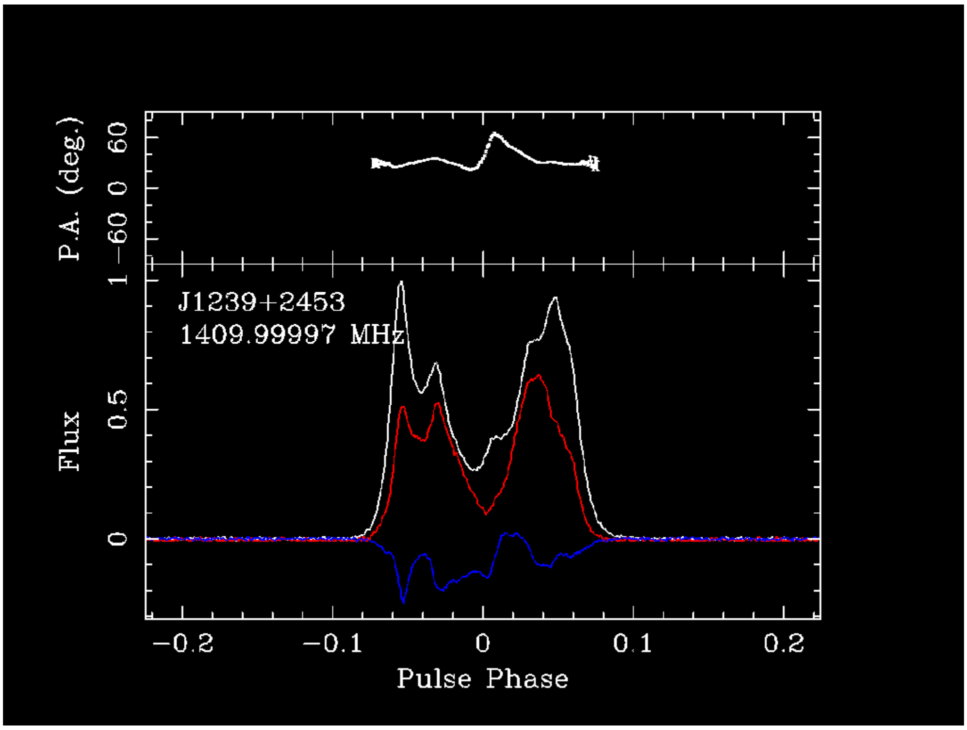
- Like magnetic declination on Earth
- Pulsar magnetic field axis not aligned spin axis

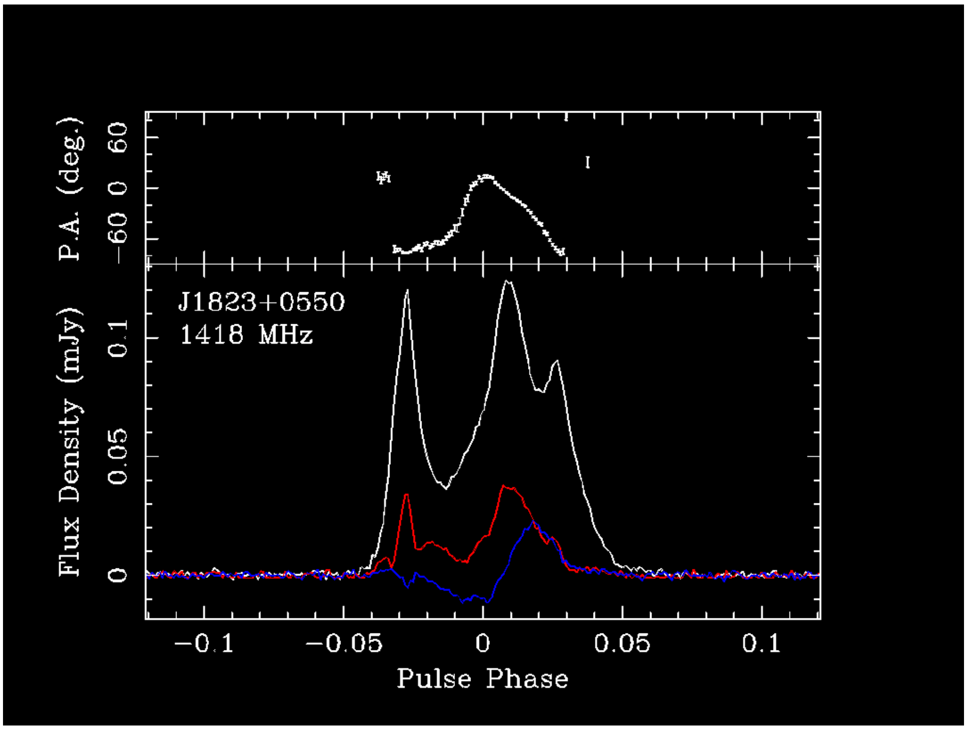


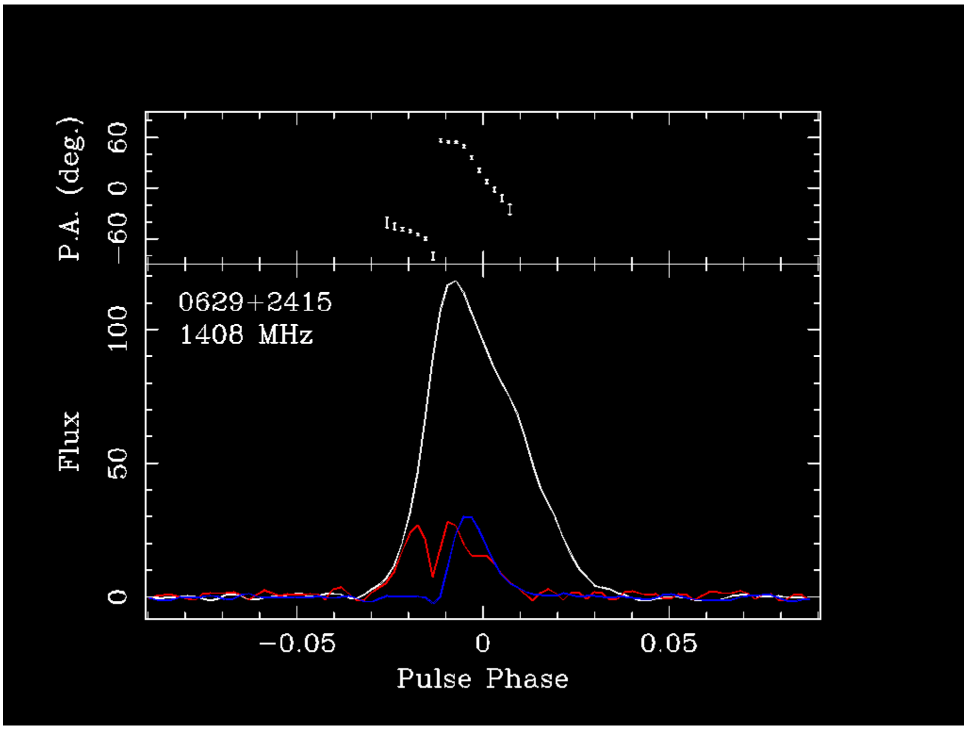


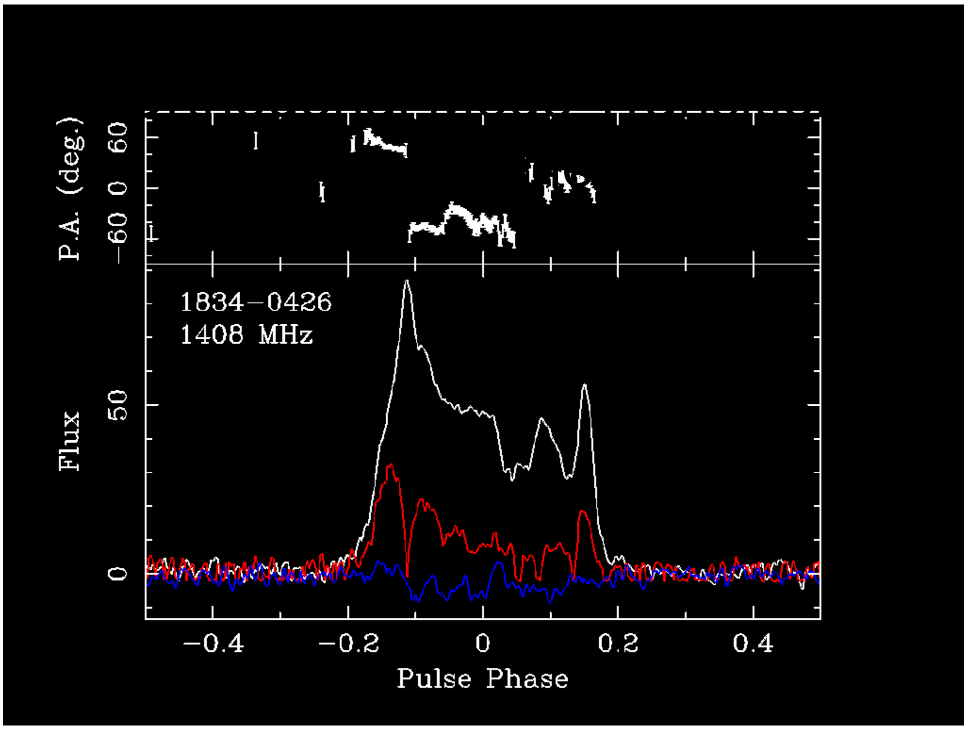
$$\tan(\psi' - \psi'_0) = \frac{\sin\alpha \sin(\phi - \phi_0)}{\sin\zeta \cos\alpha - \cos\zeta \sin\alpha \cos(\phi - \phi_0)}$$


2000
Declination (degrees east)
International Geomagnetic Reference Field (IGRF)
http://geomag.usgs.gov

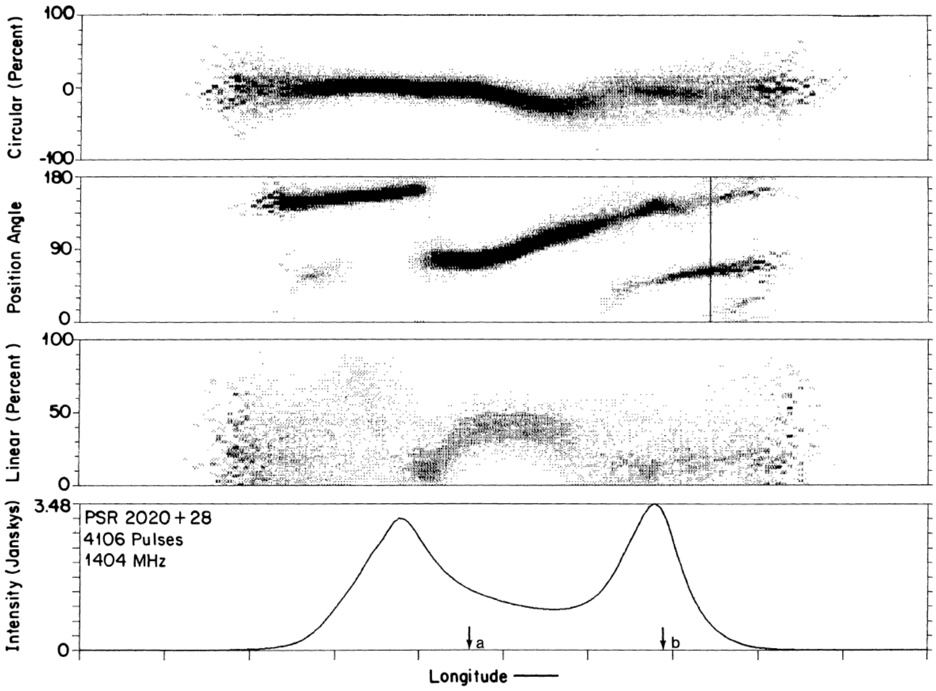

J1239+2453
1409.99997 MHz
P.A. (deg.)
Flux
Pulse Phase

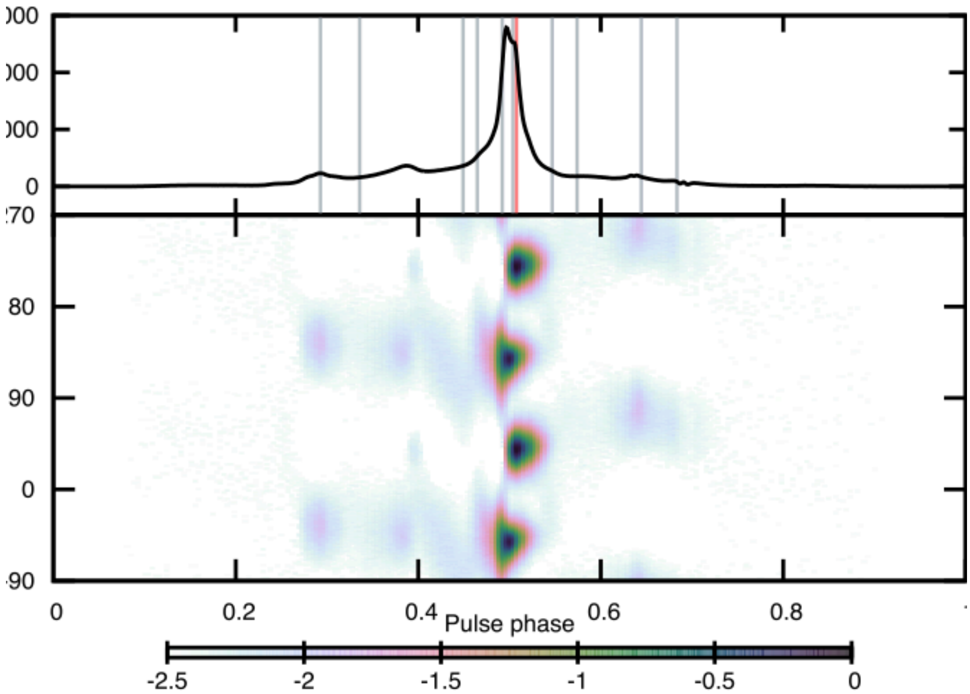

P.A. (deg.)
−60 0 60
J1823+0550
1418 MHz
Flux Density (mJy)
0 0.05 0.1
−0.1 −0.05 0 0.05 0.1
Pulse Phase


P.A. (deg.)
−60 0 60
0629+2415
1408 MHz
Flux
0 50 100
−0.05 0 0.05
Pulse Phase



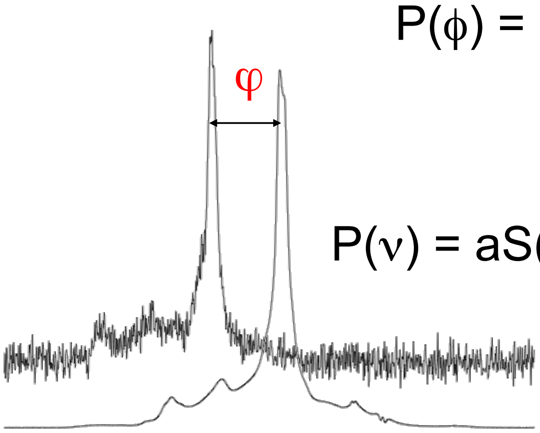
Orthogonally Polarized Modes


100
0
-100
Circular (Percent)
180
90
0
Position Angle
100
50
0
Linear (Percent)
3.48
0
Intensity (Janskys)
PSR 2020+28
4106 Pulses
1404 MHz
a
b
Longitude


0
80
90
0
0
0.2
0.4
0.6
0.8
Pulse phase
-2.5
-2
-1.5
-1
-0.5
0

High-precision Pulsar Timing

Template Matching



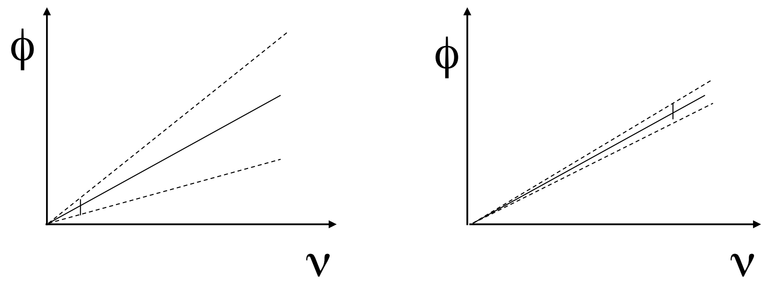
$$P(\phi) = aS(\phi + \varphi) + b$$

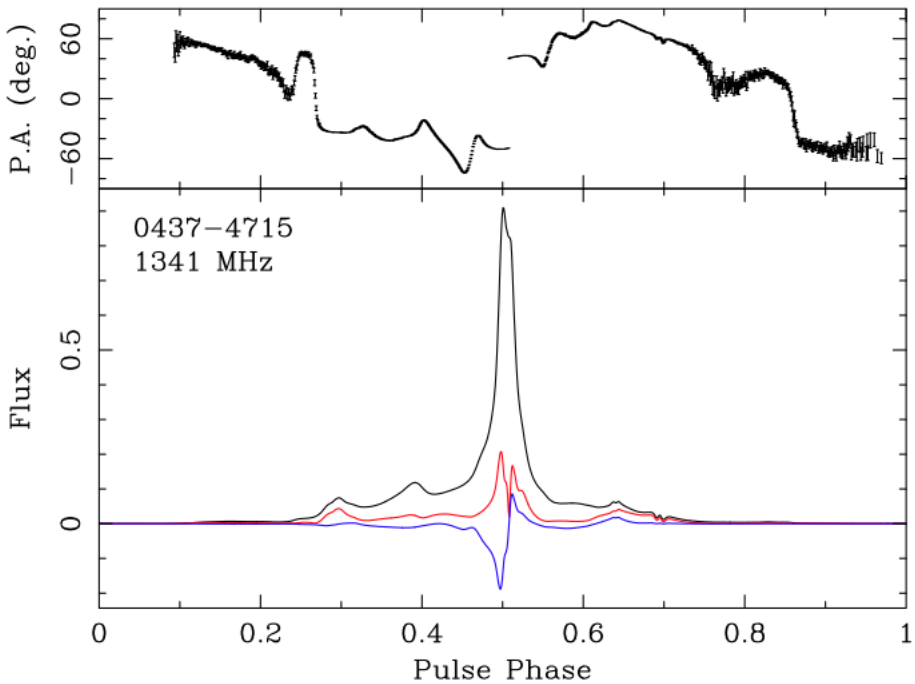
FFT

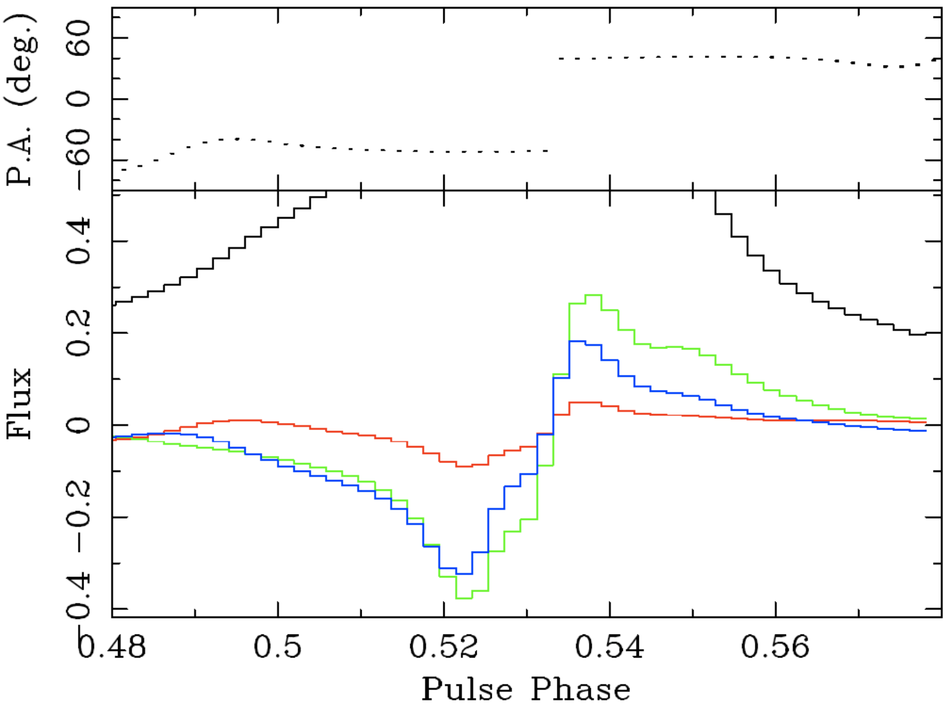
$$P(\nu) = aS(\nu)e^{-i2\pi\varphi\nu} + \delta(\nu)b$$

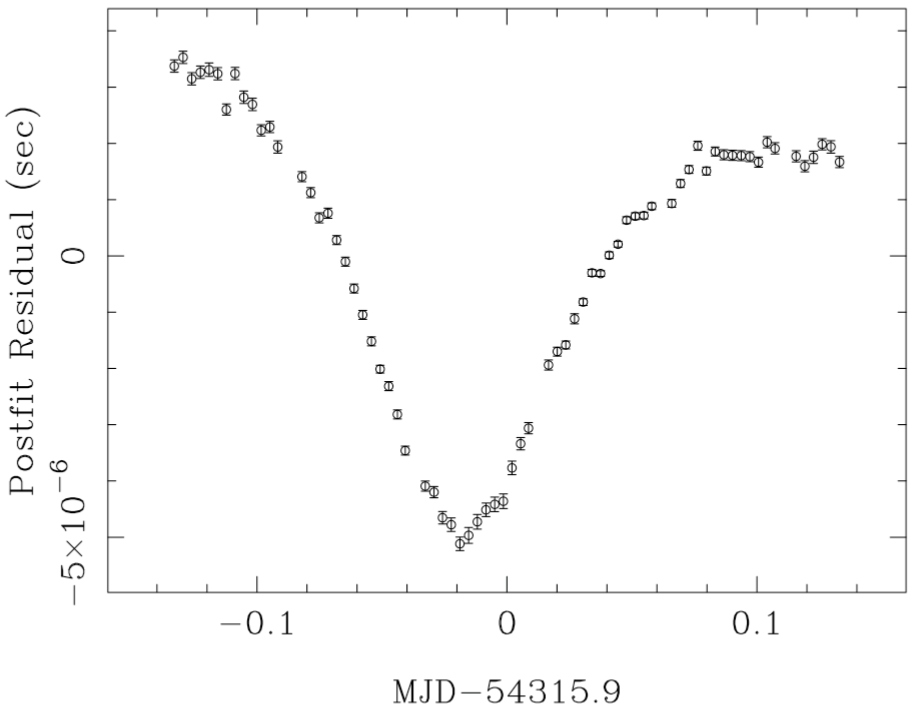
Frequency Domain

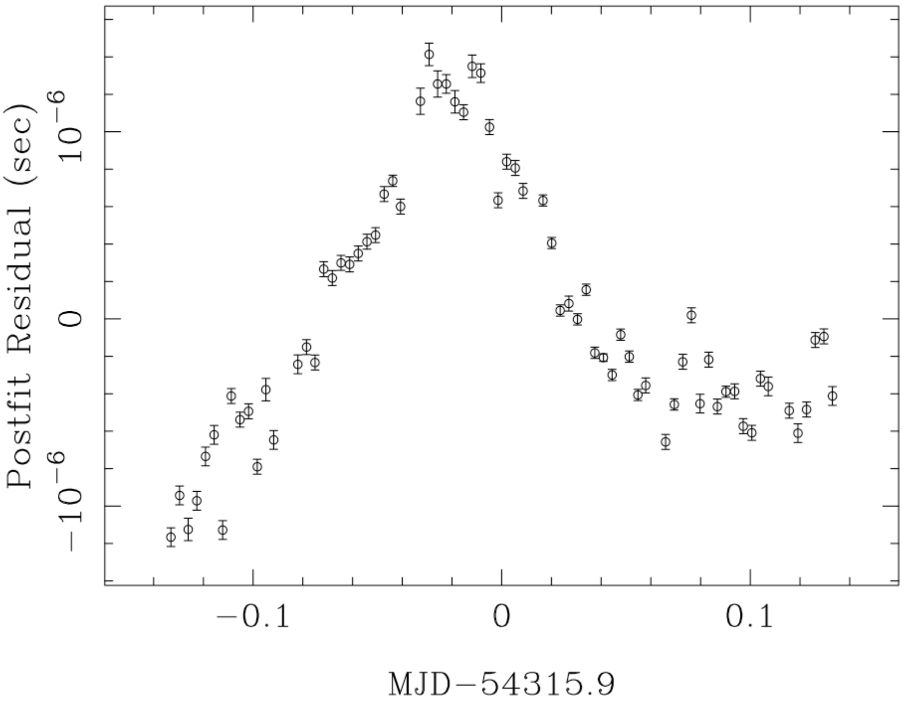
- High frequency power yields greatest constraint on slope, φ






P.A. (deg.)
60
0
−60
Flux
0.4
0.2
0
−0.2
−0.4
0.48
0.5
0.52
0.54
0.56
Pulse Phase


Postfit Residual (sec)
0
−5×10⁻⁶
−0.1
0
0.1
MJD−54315.9



Polarimetric Calibration

Calibration

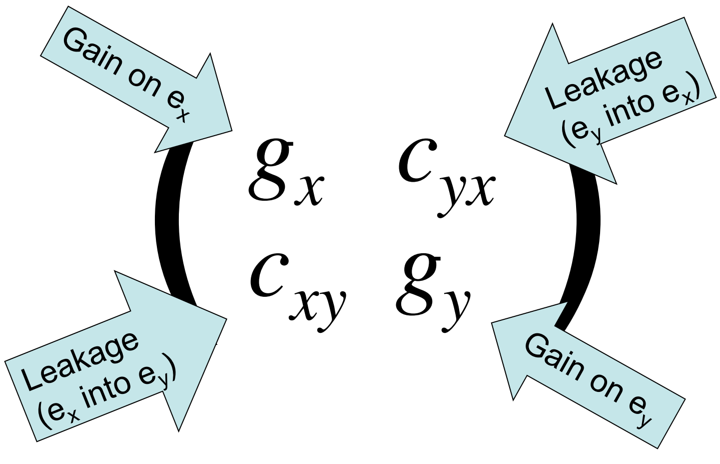
$$\boldsymbol{e}' = \mathbf{J}\boldsymbol{e}$$ Jones matrix, $\mathbf{J}$

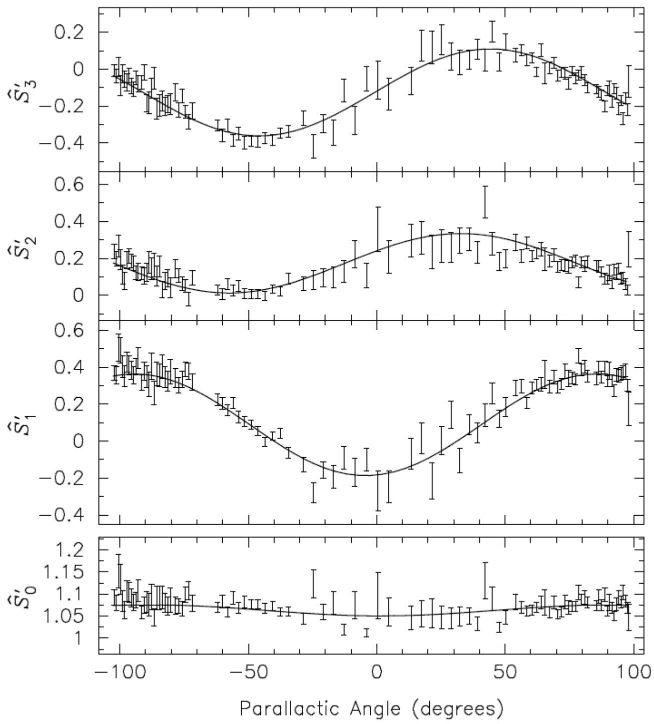
$$\boldsymbol{\rho}' = \mathbf{J}\boldsymbol{\rho}\mathbf{J}^{\dagger}$$

$$S_i' = M_i^k S_k$$ Mueller matrix, $\mathbf{M}$

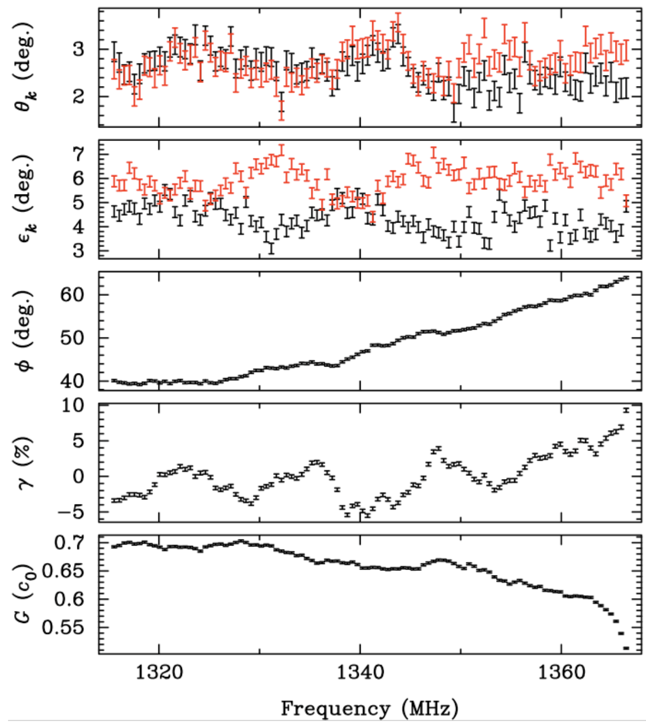
$$M_i^k = \frac{1}{2}\mathrm{Tr}(\boldsymbol{\sigma}_i\,\mathbf{J}\,\boldsymbol{\sigma}_k\,\mathbf{J}^{\dagger})$$

Jones Matrix





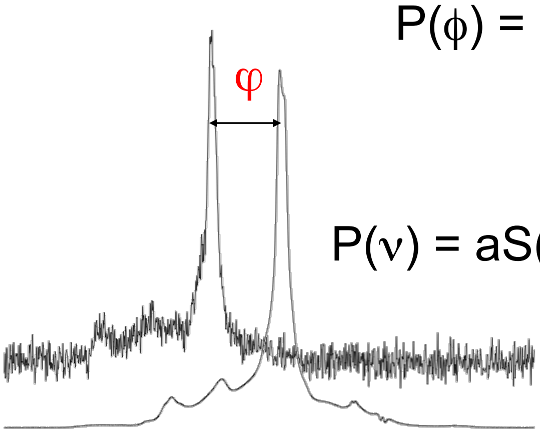
$\hat{S}'_3$
$\hat{S}'_2$
$\hat{S}'_1$
$\hat{S}'_0$
Parallactic Angle (degrees)



θ_k (deg.)
ϵ_k (deg.)
ϕ (deg.)
γ (%)
C (c_0)
Frequency (MHz)

Matrix Template Matching

Template Matching



$$P(\phi) = aS(\phi + \varphi) + b$$

FFT

$$P(\nu) = aS(\nu)e^{-i2\pi\varphi\nu} + \delta(\nu)b$$

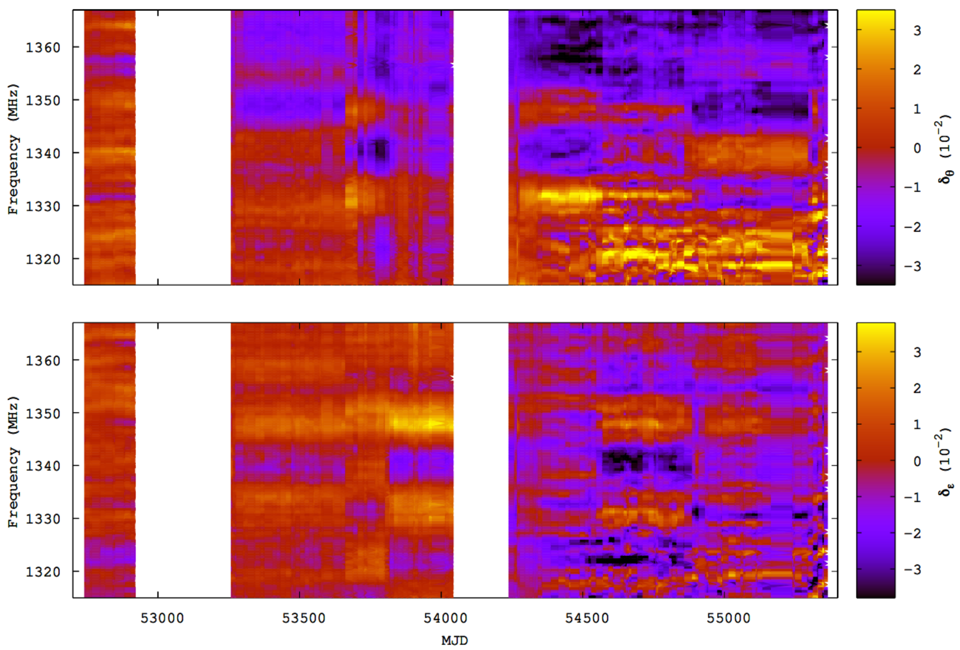
Matrix Template Matching

- Replace scalars with matrices

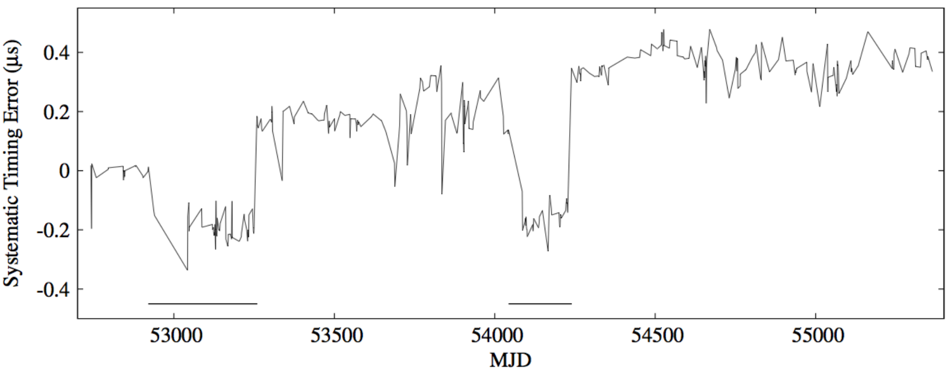
$$P(\nu) = aS(\nu)e^{-i2\pi\nu\varphi}$$

$$\boldsymbol{\rho}'(\nu_m) = \mathbf{J}\boldsymbol{\rho}_0(\nu_m)\mathbf{J}^\dagger \exp(-i2\pi\nu_m\varphi)$$

- 6 new degrees of freedom
- 4 times the number of constraints



Pulsar	$\hat{\sigma}_{\varphi}$	$\hat{\sigma}_{\tilde{\varphi}}$	τ_{β} (ns)
J0437−4715	0.85	1.43	207
J0613−0200	0.92	1.46	59
J0711−6830	0.88	1.54	81
J1022+1001	0.68	1.65	282
J1024−0719	0.74	2.11	34
J1045−4509	0.88	1.48	338
J1600−3053	0.90	1.39	115
J1603−7202	0.85	1.55	142
J1643−1224	0.91	1.40	266
J1713+0747	0.85	1.58	6
J1730−2304	0.71	1.70	198
J1732−5049	0.96	1.38	185
J1744−1134	1.56	6.43	105
J1824−2452A	0.88	2.56	18
J1857+0943	0.89	1.43	124
J1909−3744	1.02	1.51	22
J1939+2134	0.95	1.49	44
J2124−3358	0.85	1.45	127
J2129−5721	1.15	1.61	211
J2145−0750	0.95	1.44	147



Arrival Time Comparison

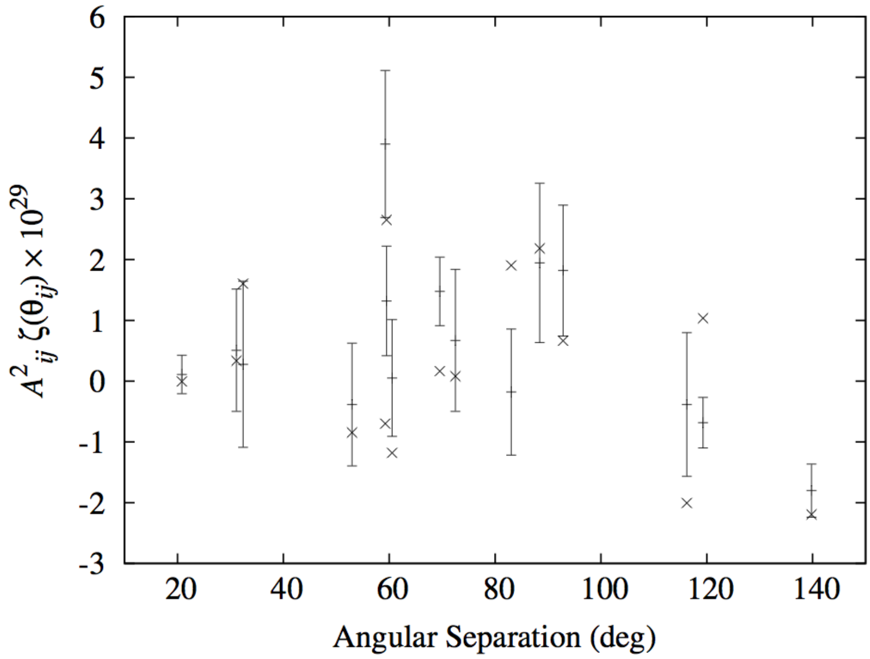
Method	σ_τ (μs)	$\chi^2/N_{\rm free}$
IFA–STI	1.9	2.2
METM–STI	1.4	2.1
IFA–MTM	0.92	1.1
METM–MTM	0.89	1.0

- 278 observations spanning ~7 years

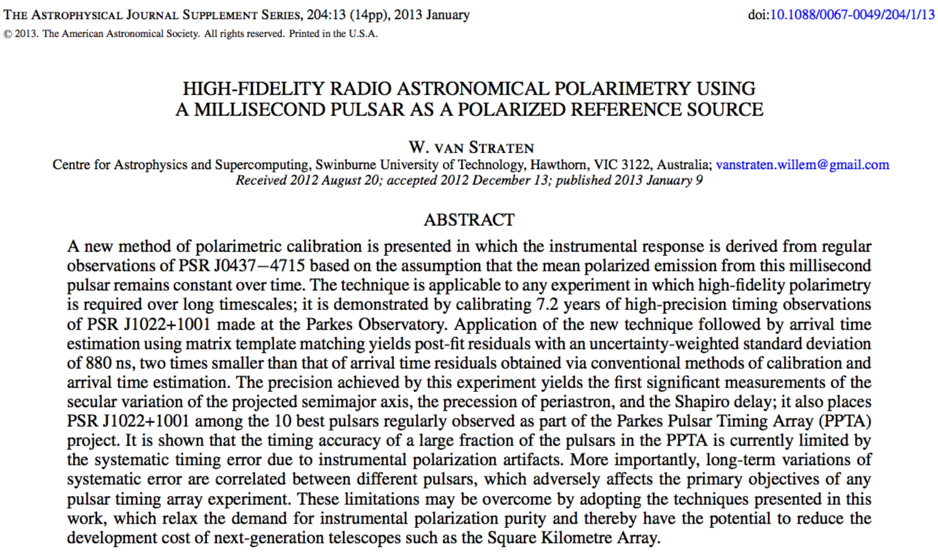
$$\mathbf{C}_\tau(f) = \mathbf{\Upsilon}\mathbf{C}_\beta(f)\mathbf{\Upsilon}^T$$

$$\Upsilon_j^k \equiv \frac{\partial \tau_j}{\partial b_k} = P_j \frac{\partial \varphi_j}{\partial b_k}$$

$$c_{AB} = \frac{\dot{\boldsymbol{\varphi}}_A \cdot \dot{\boldsymbol{\varphi}}_B}{|\dot{\boldsymbol{\varphi}}_A||\dot{\boldsymbol{\varphi}}_B|}$$



van Straten 2013, ApJS 204:13



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HIGH-FIDELITY RADIO ASTRONOMICAL POLARIMETRY USING A MILLISECOND PULSAR AS A POLARIZED REFERENCE SOURCE

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ABSTRACT

A new method of polarimetric calibration is presented in which the instrumental response is derived from regular observations of PSR J0437−4715 based on the assumption that the mean polarized emission from this millisecond pulsar remains constant over time. The technique is applicable to any experiment in which high-fidelity polarimetry is required over long timescales; it is demonstrated by calibrating 7.2 years of high-precision timing observations of PSR J1022+1001 made at the Parkes Observatory. Application of the new technique followed by arrival time estimation using matrix template matching yields post-fit residuals with an uncertainty-weighted standard deviation of 880 ns, two times smaller than that of arrival time residuals obtained via conventional methods of calibration and arrival time estimation. The precision achieved by this experiment yields the first significant measurements of the secular variation of the projected semimajor axis, the precession of periastron, and the Shapiro delay; it also places PSR J1022+1001 among the 10 best pulsars regularly observed as part of the Parkes Pulsar Timing Array (PPTA) project. It is shown that the timing accuracy of a large fraction of the pulsars in the PPTA is currently limited by the systematic timing error due to instrumental polarization artifacts. More importantly, long-term variations of systematic error are correlated between different pulsars, which adversely affects the primary objectives of any pulsar timing array experiment. These limitations may be overcome by adopting the techniques presented in this work, which relax the demand for instrumental polarization purity and thereby have the potential to reduce the development cost of next-generation telescopes such as the Square Kilometre Array.

Summary

- Instrumental polarization introduces **correlated** systematic timing error
- Long-term stability of MSP polarization can be used to calibrate instrumentation
- Matrix Template Matching can be better than quadrupling integration length

Tutorial

- Stage 1: do it yourself – python-based exploration of calibration data
  - Get data and tutorial from USB drive
- Stage 2: use psrchive – calibrate data and compare TOAs
  - Stage 2.a – introduction to psrchive